**Solution for Homework 10**

**ECE 309 Fall 2019**

**Due: November 6, 2019**

Upload an electronic copy of your answers to Moodle under HW10.

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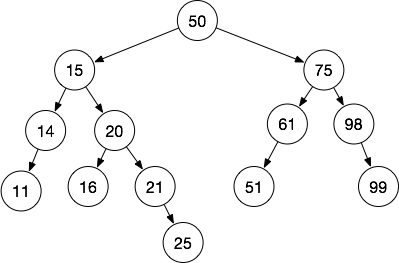
# 1. Balanced Trees

1. [40 points] For the following sequences of insertions, show the BST and AVL Tree after inserting all of the nodes. For the AVL tree, keep track of each rotation that occurs and include that as a list beside the tree. For example, for (i), you might say, “after inserting 15, rotate left at 5.”
   1. 1,2,3,4,5,6
   2. 9,8,7,6,4,5,10,11,12
   3. 20,10,25,15,30,14,25,40,26
   4. 60,65,65,30,36,64,76,68,25,52

|  |  |  |  |
| --- | --- | --- | --- |
| (i)   |  |  |  | | --- | --- | --- | | BST | AVL | AVL Rotations in order they occured:  Rotate left at 1  Rotate left at 3  Rotate left at 2 | |
| (ii)   |  |  |  | | --- | --- | --- | | BST | AVL | AVL Rotations (in the order the occurred):  Rotate right at 9  Rotate right at 7  Rotate right at 8  Rotate left at 9  Rotate left at 8 | |
| |  |  |  | | --- | --- | --- | | BST |  | Rotate right at 15  Rotate left at 10  Rotate right at 30  Rotate left at (top) 25 | |
| |  |  |  | | --- | --- | --- | | BST | AVL Tree | Rotate right 65  Rotate left at 60  Rotate left at 30  Rotate right at 60  Rotate left at 36  Rotate right at 65  Rotate right at 76  Rotatate left at lower 65  Rotate right at 36 | |

1. [20 points] For each AVL tree you drew in part (a), calculate the height and balance factor for each node.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| i.   |  |  |  | | --- | --- | --- | | **Node** | **Height** | **BF** | | 1 | 0 | 0 | | 2 | 1 | 0 | | 3 | 0 | 0 | | 4 | 2 | 0 | | 5 | 1 | -1 | | 6 | 0 | 0 | |
| Ii.   |  |  |  | | --- | --- | --- | | **Node** | **Height** | **BF** | | 6 | 3 | -1 | | 4 | 1 | -1 | | 5 | 0 | 0 | | 10 | 2 | 0 | | 8 | 1 | 0 | | 7 | 0 | 0 | | 9 | 0 | 0 | | 11 | 1 | -1 | | 12 | 0 | 0 | |
| iii.   |  |  |  | | --- | --- | --- | | **Node** | **Height** | **BF** | | 20 | 3 | -1 | | 14 | 1 | 0 | | 10 | 0 | 0 | | 15 | 0 | 0 | | 25 | 2 | -1 | | 25 (lower) | 0 | 0 | | 30 | 1 | 0 | | 26 | 0 | 0 | | 40 | 0 | 0 | |
| iv.   |  |  |  | | --- | --- | --- | | **Node** | **Height** | **BF** | | 60 | 3 | 0 | | 30 | 2 | -1 | | 65 | 2 | -1 | | 25 | 0 | 0 | | 36 | 1 | -1 | | 64 | 0 | 0 | | 68 | 1 | 0 | | 52 | 0 | 0 | | 65 (lower) | 0 | 0 | | 76 | 0 | 0 | |

1. [10 points] Consider the following AVL tree: 

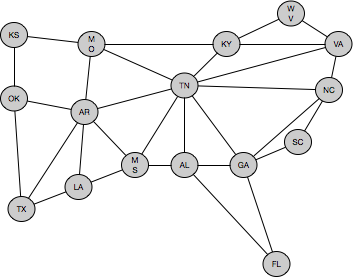
Show the tree after removing each of the following nodes (the removes compound):

* 1. 11
  2. 61
  3. 51
  4. 15

|  |  |
| --- | --- |
| (i) | Remove 11, rotate left at 15 |
| (ii) | Remove 61. No rotations needed. |
| (iii) | Remove 51, Rotate left at 75. |
| (iv) | Remove 15, replace with successor. Tree is balanced, nothing to do. |

# 2. Graph Representation

[20 points] Show an adjacency list and an adjacency matrix for the following graph.



|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Adjacency List:   |  |  | | --- | --- | | Node | Adjacency List | | AL | MS, GA, FL, TN | | AR | MO, OK, LA, MS, TN, TX | | FL | AL, GA | | GA | AL, FL, NC, TN, SC | | KS | MO, OK | | KY | VA, VW, TN, MO | | LA | AR, MS, TX | | MO | AR, KY, TN, KS | | MS | AL, AR, LA, TN | | NC | SC, GA, VA, TN | | OK | KS, AR, TX | | SC | GA, NC | | TX | LA, AR, OK | | VA | WV, KY, TN, NC | | WV | KY, VA | | TN | MO, KY, VA, AR, NC, MS, AL, GA | |
| Matrix: (every entry without a 1 is a 0)   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  | AL | AR | FL | GA | KS | KY | LA | MO | MS | NC | OK | SC | TX | VA | WV | TN | | AL |  |  | 1 | 1 |  |  |  |  | 1 |  |  |  |  |  |  | 1 | | AR |  |  |  |  |  |  | 1 | 1 | 1 |  | 1 |  | 1 |  |  | 1 | | FL | 1 |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  | | GA | 1 |  | 1 |  |  |  |  |  |  | 1 |  | 1 |  |  |  | 1 | | KS |  |  |  |  |  |  |  | 1 |  |  | 1 |  |  |  |  |  | | KY |  |  |  |  |  |  |  | 1 |  |  |  |  |  | 1 | 1 | 1 | | LA |  | 1 |  |  |  |  |  |  | 1 |  |  |  | 1 |  |  |  | | MO |  | 1 |  |  | 1 | 1 |  |  |  |  |  |  |  |  |  | 1 | | MS | 1 | 1 |  |  |  |  | 1 |  |  |  |  |  |  |  |  | 1 | | NC |  |  |  | 1 |  |  |  |  |  |  |  |  | 1 | 1 |  | 1 | | OK |  | 1 |  |  | 1 |  |  |  |  |  |  |  | 1 |  |  |  | | SC |  |  |  | 1 |  |  |  |  |  | 1 |  |  |  |  |  |  | | TX |  | 1 |  |  |  |  | 1 |  |  |  | 1 |  |  |  |  |  | | VA |  |  |  |  |  | 1 |  |  |  | 1 |  |  |  |  | 1 | 1 | | WV |  |  |  |  |  | 1 |  |  |  |  |  |  |  | 1 |  |  | | TN | 1 | 1 |  | 1 |  | 1 |  | 1 | 1 | 1 |  |  |  |  |  |  | |

# 3. ZyLab

* [10 points] ZyLab 16.20. Implement a function to calculate the balance factor of a node in an AVL tree. The function takes a node as an argument and returns the balance factor as an integer. Read the instructions in the ZyLab for more information.

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